When Can Students Misreport at No Cost?

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Strategy-proof matching: "just tell the truth"

- Centralized school choice assigns seats from the rankings that families submit, together with school priorities.
- The two leading mechanisms:
  - **Deferred Acceptance (DA)**: adopted by New York City (2003) and Boston (2005); a variant runs the US medical residency match.
  - **Top Trading Cycles (TTC)**: the main alternative (Abdulkadiroğlu and Sönmez, 2003).
- Both are strategy-proof: whatever the other students do, nobody can do better than by ranking the schools in her true order.
- So the advice to families is simple: *rank the schools in your true order of preference.*

Yet many students do not report their true ranking

- **In the lab.** In the DA treatment of Chen and Sönmez (2006), 58 of 144 rankings (40%) differ from the subject's true ranking.
  Also Li (2017), Chen et al. (2024), Kloosterman and Troyan (2026).
- **In the field.** In the Israeli psychology master's match, which uses DA, applicants often say they prefer a program *without* funding to the same program *with* funding (Hassidim et al., 2021). Some US medical students try to game the residency match (Rees-Jones, 2018).
- **The usual reading:** misreports call for a behavioral explanation: misunderstanding, expectations-based or disappointment-averse preferences, inattention, report-dependent utility, or mistakes.
  Hassidim et al. (2017), Dreyfuss et al. (2022), Chen et al. (2024), Bucher and Caplin (2021), Meisner (2023), Artemov et al. (2023).

The truth is a best report, but it need not be the only one

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- A student who submits such a ranking has misreported, but she has lost nothing.

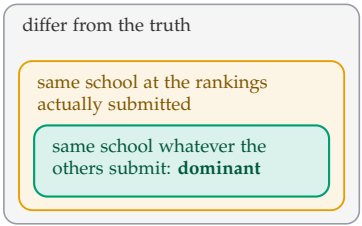
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> **The question of this paper.** When can a student misreport at no cost, getting the same school as the truth whatever the others report?
>
> This question comes before any explanation of *why* students misreport: it tells us which misreports need an explanation.

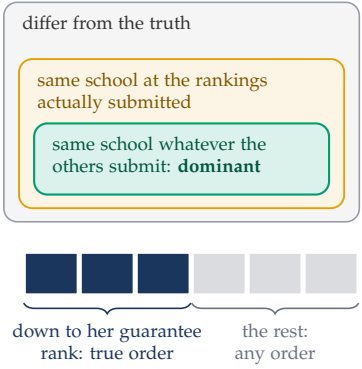
The big picture

1. **Not every false ranking is a mistake.** Some leave her school unchanged at the rankings the others *actually* submitted. A smaller set gives her the truth's school whatever the others submit: these are dominant, just like the truth.



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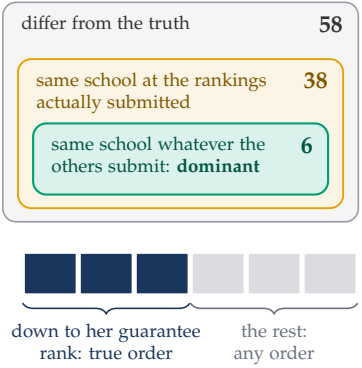
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2. **One spare seat settles the answer.** Under DA and TTC, each student has a guarantee rank: how far down her list the others can push her if she tells the truth. With one more seat than students, her dominant reports are exactly the rankings that keep her schools down to that rank first, in true order.



The same holds for every mechanism with four properties that DA and TTC share: strategy-proofness, respect for top-priority sets, top-top consistency, and feasible first choices.

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3. **In the data, the gap is large.** Chen and Sönmez (2006): 58 rankings differ from the truth, 38 of them kept the student's school, and only 6 are dominant.



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Two senses in which a misreport can be harmless

A student submits a ranking that is not her true one. Does she still get the school that the truth would give her, when the others submit …

	A … what they actually submitted?	**B** … anything else they could submit?
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- Data can check only column A: rerun the algorithm with the truth in place of her report.
- Incentives depend on A and B. This paper characterizes robust harmlessness.

How the talk answers the question

The question: when can a student misreport at no cost, whatever the others submit?

1 Setting and mechanisms

KNOWN The two leading mechanisms, and what makes a ranking dominant.

2 Menus and decisive pairs

KNOWN Menus (Barberà, 1983) decide what matters, under any strategy-proof mechanism.

NEW Decisive pairs: the comparisons she must keep. Benchmark: serial dictatorship.

3 The capacity boundary

NEW Main result for DA and TTC: the guarantee rank and one spare seat.

NEW Why they agree: it holds for every mechanism with four properties they share.

4 Without a spare seat

NEW More freedom, in other shapes.

5 Back to the data

NEW Literal misreports, realized harmlessness, and dominance in the Chen–Sönmez data.

The setting

- Students $s_1, \ldots, s_n$ and schools $c_1, \ldots, c_m$. School c has q_c seats, and $Q = \sum_c q_c$ is the total number of seats.
- Each school ranks the students by a strict priority order: siblings, walk zone, test score, lottery, ...
- Each student has strict preferences over the schools, and every school is acceptable to her.
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$Q > n$: a spare seat	$Q = n$: balanced	$Q < n$: excess demand

Deferred Acceptance (DA)

1. Each student applies to her favorite school among those that have not rejected her.
2. Each school keeps its highest-priority applicants, up to its capacity, and rejects the rest. Keeping is only tentative.
3. Repeat until nobody is rejected.

The result is the student-optimal stable matching (Gale and Shapley, 1962). DA is strategy-proof for students (Dubins and Freedman, 1981; Roth, 1982).

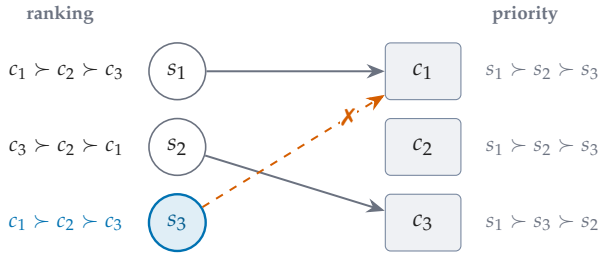


One seat at each school. Each student's ranking is on the left; each school's priority order is on the right.

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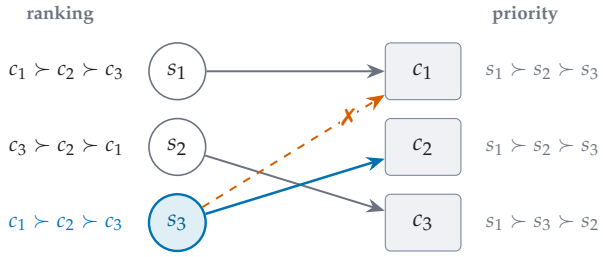


Round 1. Everyone applies to her first choice. School c_1 keeps s_1, who has higher priority, and rejects s_3.

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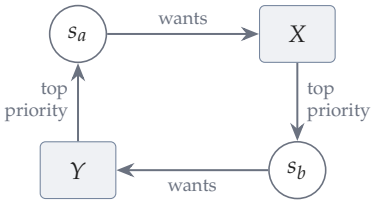
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Round 2. Student s_3 applies to c_2, which keeps her. Nobody is rejected, so DA stops:
s_1 at c_1, s_2 at c_3, s_3 at c_2.

Top Trading Cycles (TTC)

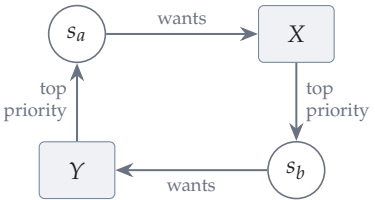
1. Each student points to her favorite school that still has a free seat.
2. Each school points to its highest-priority student still present.
3. There is always a cycle. Everyone on it gets the school she points to; remove her, and one seat of that school.
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s_a gets X and s_b gets Y: they trade their priorities.

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TTC is strategy-proof and efficient: Pápai (2000) with one seat per school (her hierarchical exchange rules extend Gale's top trading cycles, Shapley and Scarf, 1974), and Abdulkadiroğlu and Sönmez (2003) with many seats.

DA respects priorities. TTC lets students trade them.

Strategy-proofness and dominant reports

- A mechanism φ is strategy-proof if no student can ever gain by misreporting: for every true preference P_i, every ranking $\hat{P}_i$, and all rankings $\hat{P}_{-i}$ of the others,

$$\varphi_i(P_i, \hat{P}_{-i}) \;\succeq_i\; \varphi_i(\hat{P}_i, \hat{P}_{-i}).$$

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- Under strategy-proofness, and with strict preferences, this means exactly

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The object of the paper. $\mathcal{D}_i(P_i)$, the set of all dominant reports of a student whose true ranking is P_i. The truth is always in it. What else is?

Where we are

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Menus: what she can get by changing only her own ranking

How can we tell whether a ranking is dominant?

- **A known tool.** Fix the rankings of the others. Her menu is the set of schools she can obtain by changing her own ranking (Barberà, 1983).

DA example, with s_1 and s_2 fixed

s_3 submits	she gets
$c_1 \succ c_2 \succ c_3$	c_2
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A ranking is dominant $\iff$ it picks the same school as the truth from every menu she could face.

A three-student example

Which menus can the others create?

- Three students, three schools, one seat each: $Q = n = 3$.
- Focal student s_3: $c_1 \succ c_2 \succ c_3$.
- s_1 comes first everywhere.
- s_3 comes last at c_1 and c_2, but before s_2 at c_3.

Priority orders, highest first

$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
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Now bring s_3 back. She can take the empty seat, and she can take c_3 whenever s_2 sits there. Nothing else.

Her menus: c_1 and c_2 never appear together

Seat of s_1	Seat of s_2	Empty	Menu of s_3
c_1	c_2	c_3	$\{c_3\}$
c_1	c_3	c_2	$\{c_2, c_3\}$
c_2	c_1	c_3	$\{c_3\}$
c_2	c_3	c_1	$\{c_1, c_3\}$
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- Each row: how s_1 and s_2 are seated when s_3 is left out, and the menu this leaves her.
- Two-school menus arise only when s_2 sits at c_3.
- Every school is in some menu.
- But c_1 and c_2 are never in the same menu: each can reach her menu only through the one empty seat.

So reversing c_1 and c_2 costs her nothing

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Ranking of s_3	Dominant?
$c_1 \succ c_2 \succ c_3$ (truth)	✓
$c_2 \succ c_1 \succ c_3$	✓
$c_1 \succ c_3 \succ c_2$	✗
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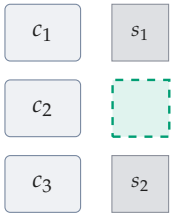
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s_3 has two dominant reports: the truth, and the ranking that reverses the order of c_1 and c_2. Both put c_3 last. Dominance fixes the *bottom* of her ranking, not the top.

Add one seat at c_3: the truth becomes her only dominant report

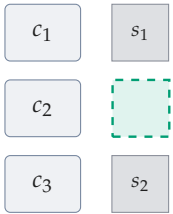
One seat each ($Q = 3$)



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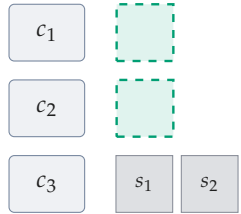
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menu of s_3: $\{c_2, c_3\}$

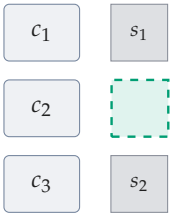
Two seats at c_3 ($Q = 4$)



menu of s_3: $\{c_1, c_2, c_3\}$

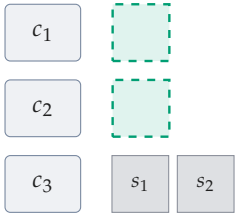
Add one seat at c_3: the truth becomes her only dominant report

One seat each ($Q = 3$)



menu of s_3: $\{c_2, c_3\}$

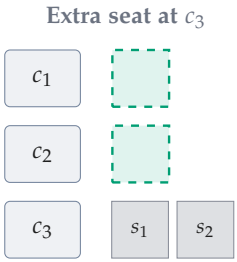
Two seats at c_3 ($Q = 4$)



menu of s_3: $\{c_1, c_2, c_3\}$

With a spare seat, s_1 and s_2 can both sit at c_3 and leave c_1 and c_2 open at the same time. Now her ranking of c_1 against c_2 decides what she gets, so reversing their order is no longer harmless.

Put the seat at c_1 or c_2 instead: the same thing happens

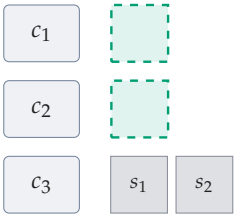


s_1 ranks c_3 first
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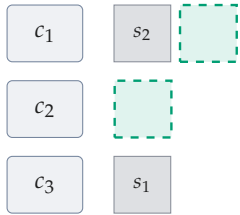
Extra seat at c_3



s_1 ranks c_3 first
s_2 ranks c_3 first

menu of s_3: $\{c_1, c_2, c_3\}$

Extra seat at c_1

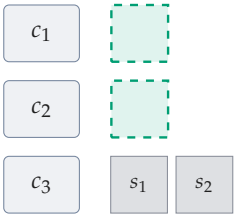


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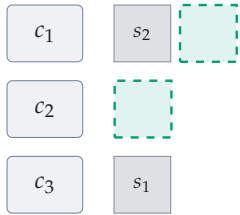
Extra seat at c_3



s_1 ranks c_3 first
s_2 ranks c_3 first

menu of s_3: $\{c_1, c_2, c_3\}$

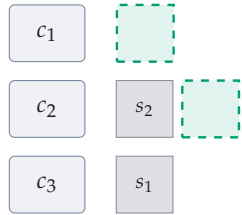
Extra seat at c_1



s_1 ranks c_3 first
s_2 ranks c_1 first

menu of s_3: $\{c_1, c_2\}$

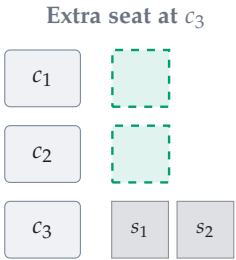
Extra seat at c_2



s_1 ranks c_3 first
s_2 ranks c_2 first

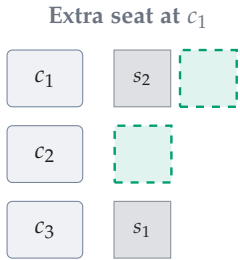
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Put the seat at c_1 or c_2 instead: the same thing happens



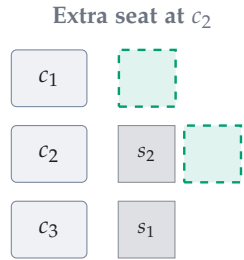
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menu of s_3: $\{c_1, c_2\}$



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Wherever the extra seat goes, s_1 and s_2 can leave c_1 and c_2 open at the same time. Then her ranking of c_1 against c_2 decides what she gets, so the truth is her only dominant report, under DA and under TTC.

Decisive pairs: the comparisons that can matter

What is the general principle behind the example?

- Write $c_{i,r}$ for her r-th school. Ranks $r < t$ form a decisive pair if some menu contains both $c_{i,r}$ and $c_{i,t}$, and nothing she likes more than $c_{i,r}$.

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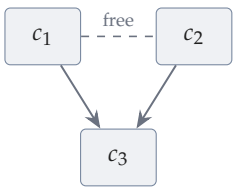
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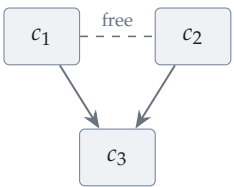
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Decisive: $(1,3)$ and $(2,3)$. Two rankings keep both: two dominant reports.

The procedure. In every menu she can face, her true favorite must stay above the other schools in it. These are the decisive pairs; the dominant reports keep all of them.

Benchmark: serial dictatorship, step by step

What does this procedure give in the simplest mechanism?

One seat per school, a fixed queue. She chooses second. True ranking: $c_1 \succ c_2 \succ c_3 \succ c_4$.

1. Her menus. The student ahead of her can take any one school.

Taken first	Her menu	Her favorite
c_1	$\{c_2, c_3, c_4\}$	c_2
c_2	$\{c_1, c_3, c_4\}$	c_1
c_3	$\{c_1, c_2, c_4\}$	c_1
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2. Decisive pairs. In each menu, her favorite must stay above every other school in it.

- First menu: c_2 above c_3 and c_4.
- Other three: c_1 above c_2, c_3, and c_4.
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3. Dominant reports keep exactly these comparisons:

$$\underbrace{c_1 \succ c_2}_{\text{true order}} \succ \underbrace{\{c_3,\, c_4\}}_{\text{either order}} \quad \Longrightarrow \quad c_1 \succ c_2 \succ c_3 \succ c_4 \quad \text{and} \quad c_1 \succ c_2 \succ c_4 \succ c_3.$$

Benchmark: serial dictatorship, any place in the queue

What if she chooses k-th, among m schools?

- The $k-1$ students ahead of her can take any $k-1$ schools: every set of $m-k+1$ schools can be her menu.



Truthful through rank k: her k favorite schools first, in their true order; the rest in any order.

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If $k < m$, the dominant reports are exactly the rankings that are truthful through rank k. If $k = m$, every ranking is dominant.

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Proposition (SD benchmark)

If $k < m$, the dominant reports are exactly the rankings that are truthful through rank k. If $k = m$, every ranking is dominant.

Those ahead can take her top $k-1$ schools, never all of her top k: k is her guarantee rank.

Where we are

1 **Setting and mechanisms**
KNOWN The two leading mechanisms, and what makes a ranking dominant.

2 **Menus and decisive pairs**
KNOWN Menus (Barberà, 1983) decide what matters, under any strategy-proof mechanism.
NEW Decisive pairs: the comparisons she must keep. Benchmark: serial dictatorship.

3 **The capacity boundary**
NEW Main result for DA and TTC: the guarantee rank and one spare seat.
NEW Why they agree: it holds for every mechanism with four properties they share.

4 **Without a spare seat**
NEW More freedom, in other shapes.

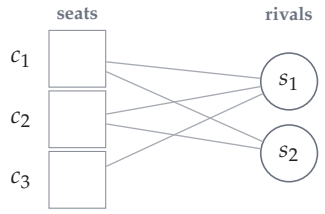
5 **Back to the data**
NEW Literal misreports, realized harmlessness, and dominance in the Chen–Sönmez data.

The guarantee rank: how far down the others can push her

What replaces her place in the queue when each school has its own priorities?

- Go down her true ranking one school at a time.
- At each step, ask: could the rivals with higher priority there fill every seat of these schools, one seat each?
- The first step at which they cannot is her guarantee rank κ_i. In serial dictatorship: her place in the queue.

s_3 in the example



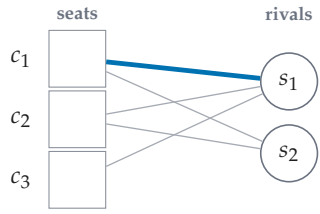
A line joins a seat to each rival who outranks s_3 at that school.

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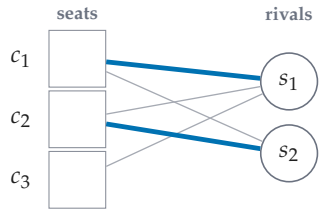
Top 1: s_1 can fill c_1.

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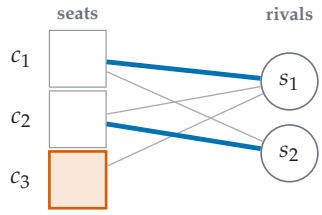
Top 2: s_1 and s_2 can fill c_1 and c_2.

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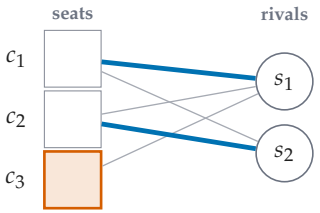
Top 3: three seats, two rivals. $\kappa_3 = 3$.

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- At each step, ask: could the rivals with higher priority there fill every seat of these schools, one seat each?
- The first step at which they cannot is her guarantee rank κ_i. In serial dictatorship: her place in the queue.
- Uses only her ranking, the priorities and the capacities (Hall's theorem).

s_3 in the example



Top 3: three seats, two rivals. $\kappa_3 = 3$.

One of her top κ_i schools always stays available. Under DA, TTC, and every mechanism with the four properties, κ_i is the worst rank the truth can give her.

Main result: the capacity boundary

Does κ_i also say how much of her ranking she must report truthfully?

Theorem 1 (Capacity Boundary), for DA and TTC

Every student ranks every school, and there are at least three schools.

(i) **Universal guarantee.** In every market, the worst school she can get by telling the truth has rank exactly κ_i, and every ranking truthful through rank κ_i is dominant.

The same theorem holds for every mechanism that, like DA and TTC, is strategy-proof, respects top-priority sets, is top-top consistent, and grants feasible first choices (e.g. Clinch-and-Trade).

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(iii) **Fails without one.** If $Q \leq n$ (no spare seat), some priority profile gives dominant reports of another shape.

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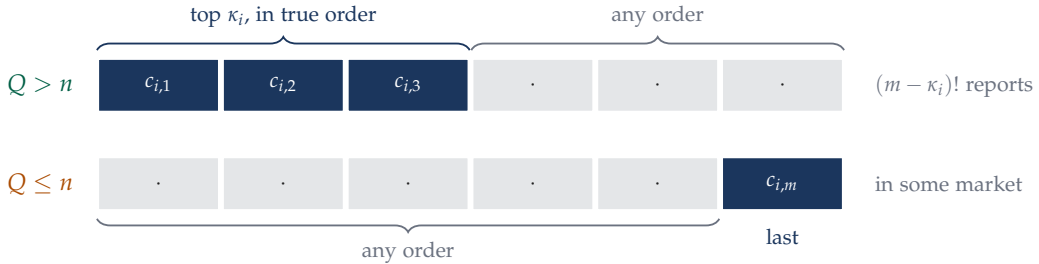
What the theorem says



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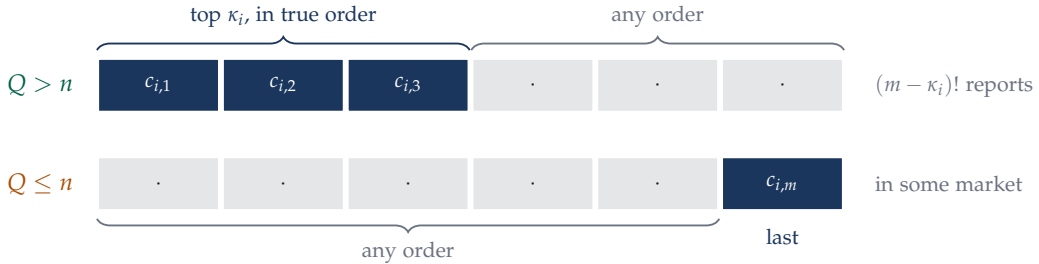


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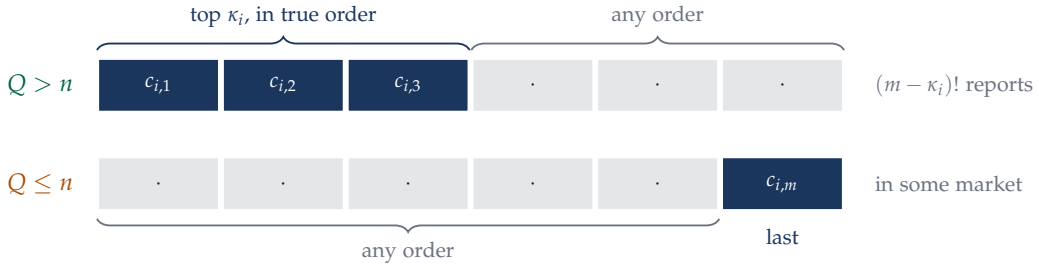
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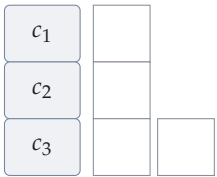
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- **Every market or some markets.** With a spare seat, this shape holds in *every* market. Without one, it fails in *some*; others keep it, e.g. with one common priority order.
- **The market, not the mechanism.** With a spare seat, every mechanism with the four properties leaves the same freedom, though DA and TTC assign different schools.

Making a pair decisive: two vacant seats

Can we construct the menus needed to establish the exact characterization?

Take two of her schools, x above y, with x at or above her guarantee rank. To make the pair decisive, we construct a profile of the others' rankings that does three things at once:

$x = c_1$, $y = c_2$; two seats at c_3



Making a pair decisive: two vacant seats

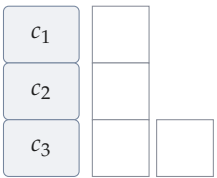
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1. **Block the better schools.** Fill every seat at the schools she likes more than x.

$x = c_1$, $y = c_2$; two seats at c_3

nothing to block: c_1 is her favorite

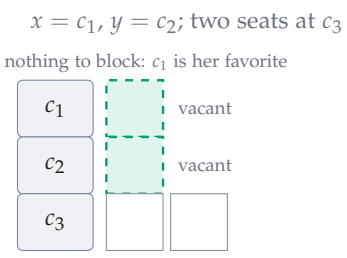


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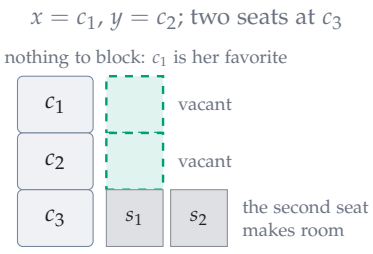


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3. **Seat everyone else.** Every rival ranks every school, so every rival ends up in some seat.

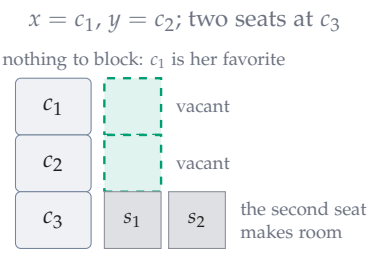


Making a pair decisive: two vacant seats

Can we construct the menus needed to establish the exact characterization?

Take two of her schools, x above y, with x at or above her guarantee rank. To make the pair decisive, we construct a profile of the others' rankings that does three things at once:

1. **Block the better schools.** Fill every seat at the schools she likes more than x.
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Musical chairs. One spare seat leaves room for two vacant seats. Without it, a second school can join her menu only through an occupied seat her priority lets her obtain.

Why one spare seat is sufficient

Does the construction work for every student and every pair?

Take any student and any pair x above y, with x at or above her guarantee rank.



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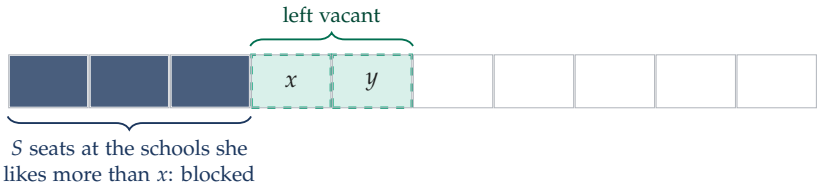
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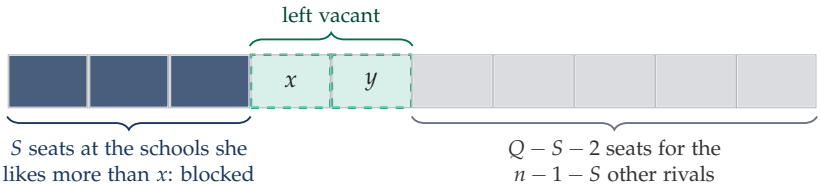
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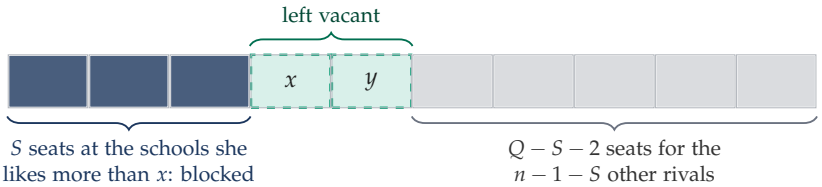
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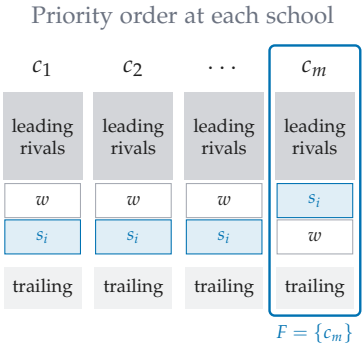


Everyone fits when $Q - S - 2 \geq n - 1 - S$, that is, when $Q \geq n + 1$.

The S cancels: one condition for every student and every pair. The resulting menu contains both schools and nothing truly better than x, so the pair is decisive.

Why the condition is sharp: the core market

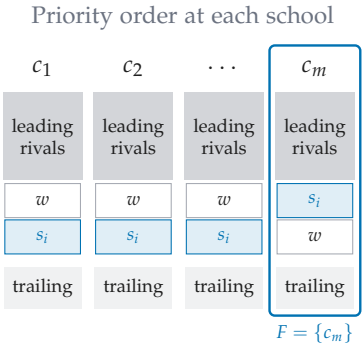
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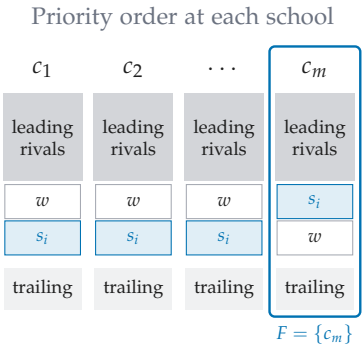
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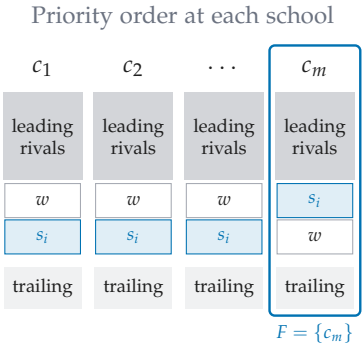
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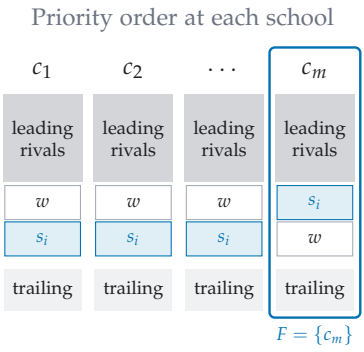
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Here $\kappa_i = m$, so the characterization would allow only the truth; yet c_1 against c_2 is never decisive. The spare seat is needed for it to hold in every priority profile.

The three-student example is the smallest core market: s_1 leads, $w = s_2$, and $F = \{c_3\}$.

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> A spare seat does the work that leaving does when lists can be short: it lets every rival be seated while two seats stay vacant.

Why DA and TTC give the same answer

The argument uses only four properties of the mechanism:

Property	What it says	What it does in the argument
(SP) Strategy-proofness	Nobody gains by misreporting.	Dominance becomes a question about menus.
(TP) Respect for top-priority sets	If those with higher priority than her cannot fill all the seats of her first h schools, she gets one of them.	Her guarantee: part (i).
(TT) Top-top consistency	A student with top priority at her first choice gets it; removing her with that seat changes nobody else's school.	Step 1, with (TP): the blockers keep the better schools.
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So Theorem 1 holds for every mechanism with these four properties.

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1 **Setting and mechanisms**
KNOWN The two leading mechanisms, and what makes a ranking dominant.

2 **Menus and decisive pairs**
KNOWN Menus (Barberà, 1983) decide what matters, under any strategy-proof mechanism.
NEW Decisive pairs: the comparisons she must keep. Benchmark: serial dictatorship.

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4 **Without a spare seat**
NEW More freedom, in other shapes.

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Balanced markets under DA: one vacant seat

Without a spare seat, which pairs are decisive?

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Easy to compute: at most n matching problems for each rank. How often does the simple shape fail? In random balanced markets with one seat per school, for about 10% of students when $n = m = 6$, and about 2% when $n = m = 30$.

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The Chen–Sönmez (2006) DA experiment

Which misreports in this experiment were harmless, and in which sense?

- 36 students and 7 schools with capacities $(3, 3, 6, 6, 6, 6, 6)$: 36 seats, a balanced market.
- Each student lives in one school's district, and each district has as many residents as its school has seats.
- Priority: own residents first; ties within each group broken by one lottery, drawn after the rankings were submitted.
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- Data: the subject-level file from openICPSR project 119341. Rerunning DA on the submitted rankings reproduces all 144 recorded assignments.

Rankings submitted before the lottery

The lottery was drawn after the rankings. What does dominance mean before the draw?

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When a queue or a priority lottery is drawn after the rankings are submitted, a ranking is dominant before the draw if and only if it is dominant after every possible draw. Probabilities and utilities play no role.

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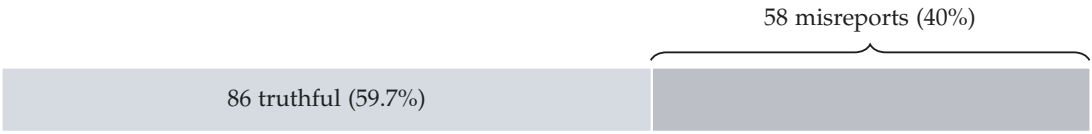
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- Her district school is guaranteed in every draw: the other residents are fewer than its seats.
- The draw that puts her first in the lottery makes every comparison down to it matter.

Classifying the 144 rankings

144 rankings

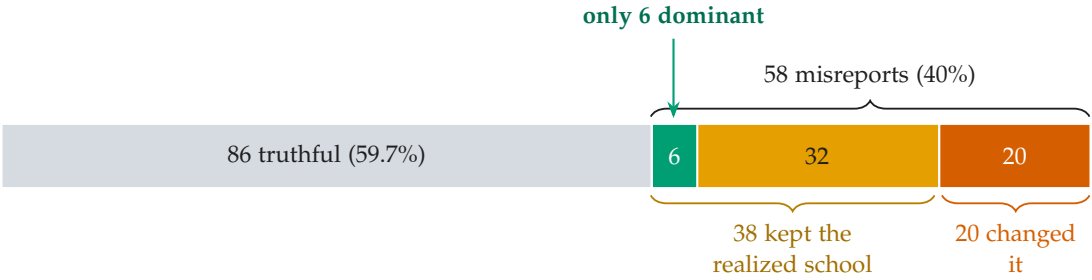
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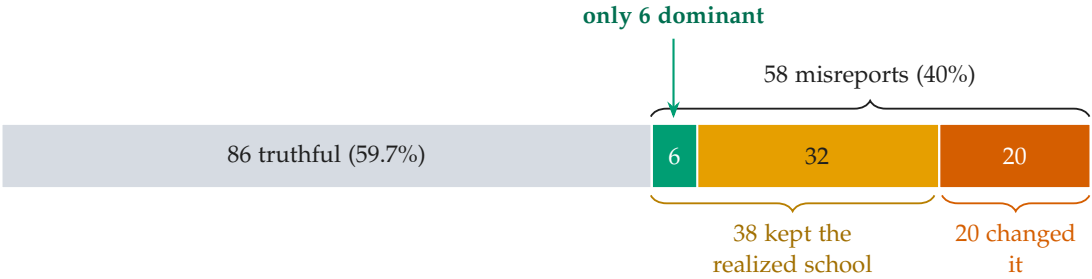
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Group	Rankings	Share of all	Share of misreports
Truthful	86	59.7%	
Nontruthful, dominant	6	4.2%	10.3%
Not dominant, same realized school	32	22.2%	55.2%
Not dominant, different realized school	20	13.9%	34.5%

Realized harmlessness overstates the freedom

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- The 6 dominant misreports only reorder schools below the district school. The mechanism gives no reason to avoid them.
- The other 52 reverse a comparison that matters for the student's school. They need a model of reporting costs, information, beliefs, or behavioral error.
- The published count of misrepresentations is 52: Chen and Sönmez (2006) count a ranking as truthful when it agrees with the truth down to the district school. In this design, that convention coincides exactly with the comparisons that matter for incentives.

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- **A sharper prediction** with a randomized default order on the screen (supplement): with a spare seat, the chance that she ranks $c_{i,t}$ above $c_{i,r}$ is

$$\tfrac{1}{2}\,\Pr(\kappa_i < r),$$

  the same for every worse school t, and weakly rising in r.

Related literature: compared with Li (2017)

Li (2017) also studies deviations from a dominant strategy. How do the two papers differ?

	Li (2017)	**This paper**
Question	When is a dominant strategy easy to see?	Which non-truthful rankings are dominant?
Idea	Obvious strategy-proofness: seeing it needs no contingent reasoning.	Decisive pairs, guarantee rank, spare seat.
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Complements. Li explains why dominant strategies can be hard to see, and how to make them obvious; this paper finds which misreports need that explanation.

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In data: 58 misreports in Chen–Sönmez, only 6 dominant.	Misreport rates (Chen and Sönmez, 2006; Li, 2017; Hassidim et al., 2017).	52 of 58 need an explanation; realized checks overstate the freedom about sixfold.

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Thank you!

Backup slides

All three students in the example • Overlapping blockers • Choosing the blockers under TTC • The last two seats and part (iii) • DA menus and realizable matchings • Dominant reports in the three cases • A better priority • Dominance before a lottery • Large markets • Choosing among dominant reports • Elicitation • Other experiments • What the supplement covers • References

The three-student example: all three students

"After" adds a second seat at c_3. Entries for s_1 and s_2 hold for all six of their true rankings. DA and TTC agree.

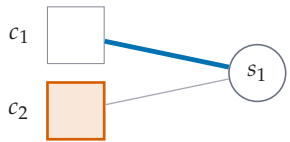
	s_1: first everywhere	s_2: second at c_1, c_2; last at c_3	s_3: last at c_1, c_2; second at c_3; true ranking $c_1 \succ c_2 \succ c_3$
Guarantee rank, before $\rightarrow$ after	$1 \rightarrow 1$	2 or 3 $\rightarrow$ 2	$3 \rightarrow 3$
All three schools in one menu?	always $\rightarrow$ always	never $\rightarrow$ sometimes	never $\rightarrow$ sometimes
Dominant reports before	her favorite first, the other two in either order	the truth only	the truth, and $c_2 \succ c_1 \succ c_3$
Dominant reports after	her favorite first, the other two in either order	the truth only	the truth only

- A full menu is not enough: s_1 always has one, yet her ranking below her favorite never matters.
- A full menu is not needed: s_2 never has one before the extra seat, yet only the truth is dominant.
- What matters is one menu per comparison, down to the guarantee rank.

Overlapping blockers: one rival takes only one seat

Three students, four schools with one seat each ($Q = 4 > n = 3$). Student s_3: $c_1 \succ c_2 \succ c_3 \succ c_4$.

Case A: only s_1 outranks s_3 at c_1 and c_2



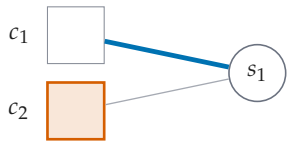
s_1 cannot fill both seats: $\kappa_3 = 2$.

Dominant: $c_1 \succ c_2 \succ \{c_3, c_4$ in any order$\}$.

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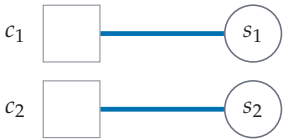
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Case B: s_1 outranks s_3 at c_1, s_2 at c_2

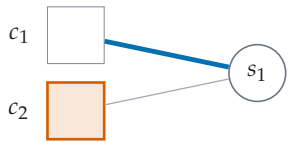


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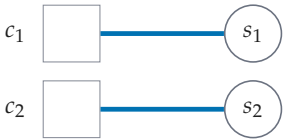
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Case B: s_1 outranks s_3 at c_1, s_2 at c_2



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Each school has *one* blocker in both cases. Counting school by school is not enough: the guarantee can come from a set of schools that the same rivals cannot fill at once.

Choosing the blockers under TTC

c_1, c_2: one seat; c_3: two seats.
Student s_3: $c_1 \succ c_2 \succ c_3$.

$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
s_1	s_3	s_3
s_2	s_1	s_1
s_3	s_2	s_2

$\kappa_3 = 2$; c_1 alone can be blocked.

- Let s_2 fill c_1 (she ranks it first), and let s_1 rank c_3 first. Under DA this works.

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Lemma (protected covering). Any blockable set of schools can be filled by rivals who, taken in a suitable order, each have the highest priority at their school among the students still present. They get these schools under every mechanism with the four properties, whatever the others report.

The last two seats in a core market

In a market with two seats, where s_i and w come before every other student everywhere:

(a) Both seats at one school c: her menu is $\{c\}$.

(b) Seats at a and b, and w comes first at both: her menu is the one of a, b that w ranks lower.

(c) Seats at a and b, and s_i comes first at both: her menu is $\{a, b\}$.

(d) s_i first at a, w first at b: her menu is $\{a\}$ if w ranks b above a. If w ranks a above b, it is $\{a, b\}$ when the mechanism lets the two exchange, and $\{a\}$ otherwise. DA and TTC always let them exchange.

Part (iii) for any mechanism with the four properties. With $m \geq 4$, take $F = \{c_{m-1}, c_m\}$: the pair $(m-1, m)$ is decisive and $(1, 2)$ is not. With $m = 3$, two markets that differ only at c_2 cover the two cases: the mechanism lets the two exchange there, or it never does.

Menus under DA and realizable matchings

- Fix the rivals' rankings, remove s_i, and run school-proposing DA among the rivals: matching η^{-i}. Then

$$c \in M_i(\hat{P}_{-i}) \iff |\eta^{-i}(c)| < q_c \text{ or } \exists\, s_j \in \eta^{-i}(c) \text{ with } s_i \succ_c s_j$$

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- **Lemma (realizable matchings).** If $Q \geq n$, every matching that seats all the rivals is produced by some complete rankings of theirs, and it is then the unique stable matching. Conversely, every realizable matching seats all rivals.
- Proof idea: each rival ranks her own school first and a school with an empty seat second. The matching is stable, and by the rural hospitals theorem (Roth, 1986) it is the only stable one.

Dominant reports under DA in the three cases

Case	Matchings of the rivals that can occur	Decisive pairs	Dominant reports
$Q > n$	Every way of seating all the rivals	Every pair (r, t) with $r \leq \kappa_i$ (Theorem 1)	Exactly the rankings truthful through κ_i
$Q = n$	Every way of seating all the rivals; one seat stays empty	Pairs with a free rival whom s_i outranks (Proposition 5.3)	Include the rankings truthful through κ_i; can fix only the last school, or nothing
$Q < n$	Matchings that fill every school and pass a graph test	A test on those matchings	Include the rankings truthful through κ_i; bounds

The case $Q < n$ is in Section S1 of the supplement. There, two easier sets of pairs bound the decisive pairs; in random small markets, the smaller set was exact in 97.7% of markets and the larger one in 72.4%.

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- In the example, if s_3 had the lowest priority at c_3 too, she would always get the school the others leave: all six rankings would be dominant.

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How often? In random balanced markets with one seat per school, the dominant reports differ from "truthful through κ_i" for about 10% of students when $n = m = 6$, and about 2% when $n = m = 30$ (supplement).

Dominance before a lottery: proof idea

- **If dominant in every state:** it gives the same school as the truth in every state, against every ranking of the others, so the same expected utility. The truth is best in every state, so the ranking is dominant before the draw.
- **If not dominant in some state ω^*:** some rankings of the others make it worse than the truth in ω^*, and never better in any state. Since ω^* has positive probability, it is strictly worse on average.
- So the dominant reports before the draw are the linear extensions of the order generated by the decisive pairs of all states together.

District schools. The district school e is guaranteed in every draw, so no pair below it is ever decisive. In the draw that puts her first in the lottery, every pair (r, t) with r at or above e is decisive (Proposition 5.3).

How much of a ranking is free in large markets?

- DA with n students, $n+1$ schools with one seat each, and priorities drawn independently and uniformly at every school (supplement, Section S9).
- The guarantee rank divided by n converges to Z with

$$\Pr(Z > x) = e^{-x} \quad (0 \leq x < 1), \qquad \Pr(Z = 1) = e^{-1}.$$

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- In the free tail: about 36.8% of adjacent comparisons and 26.4% of all pairwise comparisons.
- Under a uniformly random default order, the cheapest dominant report is nontruthful with probability tending to $1 - e^{-1}$.
- By Theorem 1(ii), the same holds for TTC. Correlated priorities shorten the truthful segment.

Which dominant report does a student submit?

- Suppose writing a ranking has a small cost, for example moving schools on a screen (supplement, Section S5).
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- If the cost is the distance from a default order, then with a spare seat the cheapest dominant report copies the truth through κ_i and the default below it.
- Predictions: with a spare seat, a better priority or an extra seat makes the submitted ranking weakly less likely to be the literal truth; and with a randomized default,

$$\Pr\big(c_{i,t} \text{ ranked above } c_{i,r}\big) = \tfrac{1}{2}\Pr(\kappa_i < r).$$

When can a mechanism stop asking?

- When the dominant reports are the rankings truthful through some rank, the mechanism needs only her top schools, in order. It can fill in the rest itself without changing her school.
- The supplement (Section S6) gives a version of DA that asks for the favorite remaining school, one at a time, stops when it is safe, keeps truthful answers dominant, and produces the usual DA matching.
- Related: Drummond and Boutilier (2014) on interview minimization; Feng and Liu (2024) on informational size in school choice.

Other experiments: serial dictatorship

- Li (2017): subjects rank four prizes before the draw that fixes their positions in the queue. Repeated sample: 209 misreports; 132 keep the realized assignment; 80 are dominant given the realized position; none is dominant before the draw.
- Hakimov et al. (2023): 12 students, two seats at each of six universities; students must pay to learn their own preferences. Revealing the current menu at the student's turn sharply redirects information purchases away from universities that are already full.

> Same message as in Chen–Sönmez: many more misreports keep the realized assignment than are dominant before the draw (132 against none for Li; 38 against 6 for Chen–Sönmez).

Related literature: the full table

Known	This paper
Menus (option sets) and pivotal voters: Barberà (1983). Dominant strategies type by type, for voting rules: Arribillaga and Massó (2016).	The tool. New: the joint question, which two schools can be in her menu together, with nothing better: the decisive pairs.
Dominant reports with short lists: Kumano and Watabe (2011, 2012). Safe sets: Decerf and Van der Linden (2021). Which schools a student can receive: Decerf et al. (2024).	Complete lists: the guarantee rank (building on Decerf et al.), the exact answer, the boundary at one spare seat, and why DA and TTC agree.
Menus under DA: Gonczarowski et al. (2026), Katuščák and Kittsteiner (2025).	Used to describe balanced markets.
Complete lists in one-to-one matching: Sirguiado and Torres-Martínez (2024). There, a spare seat restores a different incentive result.	School choice, with many seats per school: which rankings are dominant.
Why students misreport: Hassidim et al. (2017, 2021), Rees-Jones (2018), Dreyfuss et al. (2022), Chen et al. (2024), Bucher and Caplin (2021), Meisner (2023), Artemov et al. (2023), Kloosterman and Troyan (2026).	Which misreports need an explanation at all: in Chen–Sönmez, 52 of the 58.

What the supplement covers

S1 Excess demand: realizable matchings, decisive pairs, and bounds
S2 Comparative statics: better priorities and extra seats
S3 More on balanced markets: magnitudes, and another DA–TTC difference
S4 Serial dictatorship and the timing of information
S5 Selection within the set of dominant reports
S6 Preference elicitation: when a mechanism can stop asking
S7 Further experiments: Li (2017); Hakimov et al. (2023); more on Chen–Sönmez
S8 Supplementary proofs
S9 Large markets: how much of the ranking is free

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